3rd Edition

THE Dyscalculia Toolkit

Supporting Learning Difficulties in Maths

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Introduction

This book is for teachers who are looking for practical ways to help pupils who struggle with numeracy. It is aimed mainly at primary teachers who do not have a specialist background in either maths or special needs.

I use the word 'teacher' loosely to mean anyone who supports children in their learning. Parents, for example, are ideally placed to use the practical activities and games in this book to promote maths as a practical subject full of patterns and puzzles, and therefore full of interest and fun. I hope the ideas in this book will be of interest to any adults who support pupils in junior school and in the early years of secondary school, no matter whether the adults in question are classroom teachers, teachers of numeracy, teaching assistants, parents or specialist staff in special needs departments. Because the suggestions presented here are designed to promote understanding and to help learners make mathematical connections, the ideas in this book can be used to teach the basic principles of numeracy to any learner.

What's new in this edition?

- Many more games. There are now 50 games in total, including twice as many games as previously provided in the section on basic calculation strategies (Section 2).
- Ten videos, with a total running time of 48 minutes, created specially for this new edition. On the CW you can find separate videos with commentary for each of the four sections of the book, plus six short stop-motion silent movies demonstrating a selection of games from every section.
- New pupil tracking sheets, organised by teaching points and by section. The tracking sheets can be edited before being printed off.
- More downloadable teaching materials. There are now 70 pages of resources, ready to be printed off for immediate use. They comprise 18 pages of teaching resources, 18 pages of activity sheets, 20 game boards, 6 pupil tracking sheets and 10 pages of summary tables to help readers identify which activities and games are designed to target different numeracy topics. The new material available for the first time in this edition includes teaching resources, activity sheets and game boards as well as the tracking sheets mentioned above.
- Easier access to the teaching resources. All the extra material accompanying the book is now accessible from the CW .

- New and improved illustrations. Extra illustrations have been added and all the existing illustrations have been re-formatted to improve consistency and clarity.
- An expanded list of recommended reading that now includes useful websites and links to various resources and suppliers.

What does this book contain?

Inside this book you will find a collection of teaching activities and games. The activities have been developed over a number of years of teaching dyslexic, dyspraxic and dyscalculic learners, either on a one-to-one basis or in small groups of pupils who have been withdrawn from lessons for extra support. The activities are equally appropriate for children who have been diagnosed as dyscalculic as for those whose difficulties with number arise from other specific learning difficulties such as dyslexia or dyspraxia.

The book is organised into four sections:

- Section 1: Early Number Work with Numbers Under 10
- Section 2: Basic Calculations with Numbers Above 10
- Section 3: Place Value
- Section 4: Times Tables, Multiplication and Division.

Resources for all four sections can be found on the CW for the book.

The Appendix, which can be found on the CW, contains a summary of the more commonly used concrete materials, including an introduction to Cuisenaire rods. Because I use Cuisenaire rods so extensively in my teaching activities, and have found so many people unfamiliar with their use, I have also included on the CW a leaflet of practical ideas, written originally for parents.

The CW provides easy access to 70 pages of useful teaching resources, including masters for dot pattern cards and for digit cards, base-10 resources, number tracks with Slavonic shading, a variety of activity sheets, two summary tables to help you choose activities and games to target particular numeracy topics, new pupil tracking sheets, as well as 20 game boards for all the board games in this book.

The philosophy behind my teaching methods is to provide children with the kinds of practical experiences that will help them build sound cognitive models. Because the emphasis is on doing the maths rather than recording it on paper, you will find very few worksheets or ideas for written work in this book. What you will find instead is more than 200 teaching activities and 50 games. I have deliberately included activities that require only what can easily be found in a normal maths classroom or can easily be acquired by parents, such as counters, Cuisenaire rods, Dienes blocks, digit cards, dice, dominoes, paper and pencils. There is no need to buy special equipment, or commercial games and resources that tend to target only a single topic. My activities are simple to set up and most are ready for immediate use with individual pupils or small groups; others just require copies of the games boards or activity sheets from the CW .

Why is there such a strong emphasis on games?

I invariably use a lot of games in my teaching, and not simply because they are fun. Provided that the games are carefully chosen, or carefully designed, to target only a single mathematical idea at a time, games furnish pupils with the opportunity and the incentive to practise the specific techniques that we want them to acquire, allowing the ideas to become habitual and gradually more fluent. For example, having taught pupils the complement facts of 10 – the five number bonds 5 + 5, 4 + 6, 3 + 7, 2 + 8 and 1 + 9 – by allowing the pupils to manipulate concrete materials as they explore these numerical relationships in an active and practical way, pupils will still need plenty of practice using these five facts before they can become absolutely secure. That is why you will find four games, as well as several activities, that specifically target the complement facts of 10. Learners with specific learning difficulties tend to need much more repetition and rehearsal than their peers, spread over a longer period. But there are only so many times someone can recite the facts, or complete worksheets featuring them, without boredom setting in.

In parallel with work on key facts, I regularly and explicitly teach pupils reasoning strategies about how to derive an unknown fact from a known and practised fact, such as how to find various steps of a multiplication table from the three key facts for that table. A worksheet is likely to give only mechanical practice in producing the answer because it has been designed as a way of testing automatic recall, whereas a well-designed game will practise the necessary steps required to reach an answer through logic and deduction.

As a teacher, I am constantly aware of the need to contrive situations in which the facts and techniques that my pupils need to master can be rehearsed in as many different ways as possible. Games are extremely valuable in this context because children are naturally motivated to spend time playing games over and over again and rarely notice how much learning and reinforcing is taking place while they are actively and productively engaged in play. As well as being more enjoyable, games are more powerful than worksheets because each time a game is played different challenges might be presented, in a different sequence, leading to new considerations and different outcomes, all of which provides for a much more varied, stimulating and active learning experience.

Purposeful activities and targeted games are central to my approach to teaching learners who have difficulties with basic maths. The activities I specify in this book are never intended to simply back up paper-and-pencil techniques or abstract methods. Neither are my games ever intended as just a bit of fun to fill in the spare time at the end of a lesson. All the activities and games that I design are invented to provide the actual learning experience for a variety of very specific maths topics.

How to use this book

Please do not feel that you ought to start at the beginning of the book and work through to the end, or even to keep to the sequence in which the ideas are presented. You should feel free to pick and choose activities, depending on your pupils and your knowledge of their particular areas of difficulty. Some activities may need to be repeated often, or revisited at regular intervals; others may be valuable to try only once for particular pupils, or not

at all. When activities naturally follow on from each other, the text clearly signals the fact. Some activities may need to be preceded by others from another section; for example, some understanding of place value (Section 3) is required before attempting some of the work on larger numbers (Section 2) and before some of the work on multiplication and division (Section 4). Once you begin working closely with pupils, you will find that you are the person best placed to uncover any misconceptions or sticking points that could usefully become the focus of subsequent lessons.

Each of the four sections starts with a short overview, putting the topic of that section into context. Following the overview, you will find a summary of the main problems associated with the topic leading to a list of ideas on how to help. These summaries are presented as bullet points for ease of reference. The remainder of each section is dedicated entirely to the teaching activities and games, set out as clearly and concisely as possible with a minimum of explanatory background or theory. Printable and photocopiable resources from all four sections are provided on the CW, making the activities and games accessible and ready to use, with the minimum of preparation.

I have targeted what I know to be specific areas of difficulty and have deliberately broken down the teaching and learning into extremely small steps. Each section is loosely structured in order of difficulty, starting with concrete activities and progressing gradually through learning activities that are designed to help pupils move through the intermediate diagrammatic stage and right up to the abstract stage of calculation.

Most of the activities are designed to be teacher-led, rather than for children to work through on their own. It is important to ask lots of questions, to direct the discussion carefully, to point out any connections with previous activities and other maths topics and to encourage pupils to talk a lot about what they are doing, and why, while they are doing it. Naturally, pupils will do best in an atmosphere where mistakes are regarded as a normal, and even an instructive, part of the learning process.

Whether inside the classroom or at home, the best results will be achieved by frequent, regular, short but unhurried, sessions, each of which should include a variety of activities and topics and a sensitive balance between revision and new content. Daily sessions will soon improve pupils' attitude and will steadily boost their self-assurance, their sense of achievement and their maths performance.

The 200+ activities are each labelled according to the main teaching point they have been designed to address. A list of teaching points is also included in the instructions for each of the 50 games. The main numeracy topic addressed by each game is summarised in the first of the two tables at the end of this Introduction. The second summary table at the end of this Introduction links the activities and games in this book to a list of numeracy topics and teaching points so that you can easily find ideas to target a particular gap in a pupil's knowledge or to address a specific misconception or need. As well as appearing at the end of this chapter, both the summary tables can be found on the CW . Tracking sheets, accessible from the CW, are provided for the first time in this latest edition of *The Dyscalculia Toolkit*. Closely linked to the summary tables, the tracking sheets are designed to help you plan work for individual students or groups of pupils, and to use for tracking and recording a learner's progress. The tracking sheets can be edited before being printed off.

What is dyscalculia?

Developmental dyscalculia was first recognised in the UK by the Department for Education and Science in 2001 and defined as: 'a condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence' (DfES 0512/2001, p. 2).

There is a debate about whether true dyscalculia differs from the maths difficulties experienced by some dyslexic and dyspraxic learners, a debate I am happy to leave to the academics. What matters to me is the fact that the same sorts of intervention seem to help many pupils who are underachieving in maths, whatever label they have been given. I believe that the coming years will see a growing recognition of the particular problems and educational needs of dyscalculic learners, in much the same way as the last three or four decades have seen an increasing acceptance of the existence of dyslexia and a developing consensus about the best teaching and learning approaches for these pupils.

Research into dyscalculia is still at an early stage, but it is estimated that dyscalculia affects roughly 4–6% of the population. This equates to at least one child in any average classroom.

What are the indicators for dyscalculia?

As a teacher, you might suspect that you have a dyscalculic pupil in your class if an otherwise competent student has a surprising level of difficulty with ordinary numeric operations and relies on finger-counting, often for all four arithmetic operations, well beyond the age at which most of the others in the class have progressed to more efficient strategies. A dyscalculic learner stands out as having no 'feel for numbers' at all, no ability to estimate even small quantities, and no idea whether an answer to an arithmetic problem is reasonable or not. Memory weaknesses, both long-term and short-term, are a great handicap and result in a pupil with dyscalculia being unable to remember facts and procedures accurately, or consistently, no matter how many times they try to learn them by heart. Pupils who have dyscalculia simply cannot remember their times tables reliably, and you may find they can recall some facts one day but not the next. They are also likely to lose track of what they are doing when attempting any procedure that requires more than two or three steps. Even basic counting can be a problem for pupils with dyscalculia, especially counting backwards.

Indicators for dyscalculia include:

- an inability to subitise (see without counting) even very small quantities
- an inability to estimate whether a numerical answer is reasonable
- weaknesses in both short-term and long-term memory
- an inability to count backwards reliably
- a weakness in visual and spatial orientation
- directional (left/right) confusion
- slow processing speeds when engaged in maths activities

- trouble with sequencing
- a tendency not to notice patterns
- a problem with all aspects of money
- a marked delay in learning to read a clock to tell the time
- an inability to manage time in daily life.

What about learners with other specific learning difficulties?

A dyslexic pupil might show many of the same indicators as those listed above, because it is thought that at least half of all dyslexics also have difficulties with maths. Outside the maths classroom, you might suspect that pupils are dyslexic if they read and write much less willingly and fluently than you would expect, if they read and re-read written material with little comprehension and if their spelling is particularly weak, inconsistent or bizarre. Dyslexic learners show much greater ability and understanding when speaking than you could ever guess from looking at the scrappy and minimal amount of written work they produce. Other indicators are memory weaknesses, problems with processing auditory information, and difficulties with planning and organisation.

A typical dyspraxic pupil does not seem to have the same long-term memory problems as a dyslexic and so might be able to remember times tables facts with ease. Dyspraxia, also known as DCD (developmental coordination disorder), mainly affects motor control, which results in pupils being clumsy and uncoordinated, poor at planning and organisation, and unsuccessful at subjects like PE and sports that require balance and coordination. Dyspraxic pupils cannot process sensory information properly and are therefore forever tripping and falling, dropping and breaking things, and mislaying their belongings. In the maths classroom, dyspraxic pupils have particular difficulty in handling equipment such as a ruler, a protractor or a set of compasses, and their written work is likely to be very messy and difficult to decipher.

A pupil with attention deficit hyperactivity disorder, may signal his (and it is usually a boy) presence by being unable to stop fidgeting or to sit still, being too easily distracted by outside stimuli, having a tendency to talk and interrupt excessively, and finding it extremely difficult to stay on task and see any undertaking through to the end. I mention the condition here only because nowadays pupils with ADHD or ADD tend to come under the umbrella term of 'learners with specific difficulties'. However, pupils with attention disorders may not have any specific problems with numeracy or maths once they have found a way to manage their impulsivity and concentration difficulties.

What kind of teaching do dyscalculic learners need?

All numeracy teaching should aim to help learners build up a sound mathematical understanding of numbers and their relationships. The basis of my own teaching approach with dyscalculic learners is to concentrate on numeracy and arithmetic, starting – crucially – with a variety of versatile concrete materials that provide practical experience and strongly visual models. Once a

numerical concept has been understood at the concrete level, then, and only then, will I begin to lead the learner gradually but steadily towards some of the more abstract and symbolic methods associated with higher level mathematics.

My own view is that a set of Cuisenaire rods is indispensable for working with dyscalculic learners. I find it the best, most versatile and most powerful tool to offer learners who are struggling to build a coherent mental model of the number system. I supplement Cuisenaire rods at the lower end with discrete items, such as counters or nuggets, that can be arranged and re-arranged into dot patterns for the numbers up to 10, and at the upper end with Dienes blocks or other base-10 equipment that can combine with Cuisenaire rods for concrete modelling of 3-digit numbers. One of the great strengths of Cuisenaire is that numbers are not presented as a collection of ones, so that the learner's focus is directed away from counting and towards number relationships. See the Appendix for an introduction to Cuisenaire rods and other concrete materials.

Dyscalculic learners, just like other learners, need to be able to count properly. Counting is, after all, the foundation of all numeracy. But, as soon as counting is secure, children have to be taught calculation strategies that do not rely on counting in ones. For this reason, I recommend plenty of work on building numbers from smaller components, splitting quantities up again into smaller chunks, and recombining the component pieces once more in order to fully explore the composition of numbers and the connection between addition and subtraction. The component work that I describe in this book is less static than simply learning the number bonds because its emphasis is on performing operations on numbers and seeing quantities change as a result of whatever action is being performed.

Working with chunks, or components, rather than ones, is the only antidote to the immature – and damaging – dependence on counting that is so common in pupils who struggle with numeracy. When counting is the only strategy known to learners, they have fallen into the 'counting trap'. See the Overviews of Section 1 and Section 2 for more about this pervasive problem. The only way out of the vicious cycle is to explicitly teach learners calculation strategies based on components, i.e. chunks, and not on counting on, or counting back, in ones.

Similarly, the times tables work and the multiplication and division activities that I recommend in this book are very far removed from the all-too-common practice of giving children a list of tables facts to learn by heart, a situation that leaves many pupils without any idea about what multiplication or division mean or how to use or apply the facts they have been asked to memorise. My teaching approach to tables is based on the area model of multiplication and division, an interpretation that inherently connects multiplication with division from the very beginning and one that can be modelled with Cuisenaire rods to produce rectangular shapes that are easy to read, understand and visualise.

Problems with numeracy often go hand in hand with significant memory weaknesses. This is why simple repetition will never be a way forward for dyscalculic pupils, however hard or often they are drilled. The best way to work around learners' memory problems is to focus on only a few key facts, those that are more important or have the widest application: first allow learners to thoroughly explore and internalise the key facts; then teach them explicitly how to derive whatever other facts they might need by reasoning logically from the key facts they already know.

Visualisation is a strategy that should be explicitly taught to dyscalculic pupils as a route towards mental calculation strategies. Immediately after a session of concrete work, pupils

can be asked to close their eyes and try to recreate some of the work in their mind's eye. Diagrammatic calculation methods, such as empty number lines for addition and subtraction or the area model for multiplication and division, can be introduced as a way of recording concrete work with Cuisenaire rods and later extended to support visualising techniques, thereby creating a bridge between concrete exploration with manipulative materials and the more abstract work that is the norm in mainstream schools. The transition between concrete and abstract work is an important stage that needs to be planned for and cannot be rushed.

My final observation about what kind of teaching approach works best for dyscalculic learners is a recommendation to break down every bit of teaching and learning into the tiniest of incremental steps and not to make any assumption about what pupils already know. For example, just because a child knows, say, that five counters can be arranged into the familiar dice pattern for 5, it does not follow that the same child will know that none of the other dice patterns can be created out of exactly five counters; or just because a child has discovered that adding 1 to each of the numbers up to 10 results in the next number in the counting sequence or that taking 1 away results in the previous number, it does not follow that the same child will know how to add 1 to a 2-digit number, let alone be able to work out how many to take away from a quantity in order to leave 1. Sound numerical understanding can only develop if it rests on secure foundations at every stage. The importance of tightly focused practical activities that address only a single new idea at a time cannot be overestimated.

What's next, after working through this book?

I have written two other books published by Sage that follow on from this one, although each book is complete in itself and can be used independently of the other two. *Overcoming Difficulties with Number* is aimed at learners who are working at a slightly higher level or who have, perhaps, already worked through many of the ideas in *The Dyscalculia Toolkit*. *Overcoming Difficulties with Number* analyses some key numeracy strategies – such as bridging through 10, or learning multiplication tables through the area model – in very great detail, setting out step-by-step instructions on how to teach the strategies to learners who find the concepts difficult. The focus is on teaching for understanding while at the same time helping learners make the transition from practical exploration to more abstract and canonical calculation methods.

The Dyscalculia Resource Book is a collection of ready-to-use and printable games and puzzles, all carefully targeted to practise the crucial foundation skills – such as adding and subtracting in component chunks rather than in ones, or deriving new numeracy facts from known key facts – that children who struggle with numeracy need to master before they can make any significant progress in maths. The games and puzzles in *The Dyscalculia Resource Book* are designed to reinforce what has previously been taught at a concrete level – for example through the activities and games in this *Dyscalculia Toolkit* book – and each is accompanied by clear instructions to the supervising adult on how to manage the activity so as to maximise the learning experience.

You are welcome to contact me through my website (www.ronitbird.com) with any feedback about how your children or pupils respond to the ideas in any of my books. On my website you will also find a list of Top Ten Tips for Parents, general information and online links to do with dyscalculia, a variety of free games and teaching resources, and details of my ebooks, all of which contain many demonstration videos.

SUMMARY OF THE MAIN TEACHING POINTS ADDRESSED BY THE GAMES IN *THE DYSCALCULIA TOOLKIT*

This table is provided because the name of each game does not always reveal exactly what topic it was designed to target (unlike the main teaching points of the activities, which are spelled out by their titles).

A downloadable version of this table is available via the Companion Website

GAME	MAIN NUMERACY TOPIC/TEACHING POINT	LOCATION
Make 5	Split and recombine numbers up to 5	Section 1
Numbers Inside	Identify smaller components of larger numbers	Section 1
Collect 5s	Add numbers up to 4 + 4	Section 1
Key Components Guessing Game	The key component facts: doubles or near-doubles	Section 1
Odd and Even Collectors	Recognise odd or even up to 10	Section 1
Draw Your Race on a Number Line	Empty number line versus number track	Section 1
Race to Tell a Story	Build the same target number in different ways	Section 1
Post-It Note Subtraction	Focus on subtraction, as inverse of addition	Section 1
Cover the Numbers / Shut the Box	Split and recombine numbers up to 12	Section 1
Clear the Deck	Components of a target number up to 10	Section 1
How Many Beads? How Many Hidden?	Bridging through 5 and through 10	Section 1
Complements Number Search	Complements to 10 (i.e. components of 10)	Section 1
Complements Ping-Pong	Complements to 10 (i.e. components of 10)	Section 1
Ten in a Bed	Complements to 10 (i.e. components of 10)	Section 1
Who Has the Most Equations?	Add and subtract 1 or 2	Section 1
Polka Dots	Visualise and use reasoning for components of 12	Section 2
Regroup	Visualise and use reasoning for numbers up to 20	Section 2
It All Adds Up	Build 'teen' numbers out of smaller components	Section 2
5 and What's Left	Bridging through 5 and through 10	Section 2
Frame an Addition	Bridging through 10	Section 2
Race Along a Number Line and Bridge	Bridging through 10	Section 2
Race to the End of the Number Line	Recognise when bridging is not needed	Section 2
Frame a Subtraction	Bridging through 10 for subtraction	Section 2
Keep the Change!	Complements to 100 (i.e. components of 100)	Section 2
Double Take	Doubling and reasoning about near-doubles	Section 2
Magic 10s	Grouping tens for place value	Section 3
Race to Cover 100	Exchange and decomposition	Section 3
Four Throws to Reach 100	Place value decimal system up to 100 or 1000	Section 3
Dice and spinner games	Place value, explored actively	Section 3
Spot the Decomposition	Decomposition in subtraction	Section 3
Win Counters on a 100-Square	Place value structure of 2-digit numbers	Section 3
Race Through a 100-Square	Place value structure of numbers up to 100	Section 3
Steer the Number	Place value structure of 2-digit numbers	Section 3
Two-Digit Sequences	Sequencing 2-digit numbers	Section 3
Three-Digit Sequences (Focus on Tens)	Sequencing 3-digit numbers	Section 3
Place Value Boxes	Place value of large numbers (6 digits)	Section 3

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GAME	MAIN NUMERACY TOPIC/TEACHING POINT	LOCATION
Calculator Skittles	Place value of large numbers (4 digits or more)	Section 3
Jump 10	Adding 10 to a number	Section 3
The Six-Card Rounding Game	Place value in 2-digit numbers	Section 3
The Rounding Challenge	Place value in 2-digit or 3-digit numbers	Section 3
Don't Walk if You Can Take the Bus	Derive multiplication facts from key tables facts	Section 4
Mouse Tables	Learn and practise a chosen times table	Section 4
Self-correcting tables cards Game 1	Learn and practise a chosen times table	Section 4
Self-correcting tables cards Game 2	Finding the questions and focusing on division	Section 4
Self-correcting tables cards Game 3	Matching questions to answers in a chosen table	Section 4
Self-correcting tables cards Game 4	Snap, matching multiplication to division	Section 4
Multiples from the 1–6 Times Tables	Practise tables facts up to 6 × 6	Section 4
Products in a Row	Connect pairs of related multiplication tables	Section 4
Factors	Division: find factors of given multiples	Section 4
Areas on a Grid	The area model of multiplication and division	Section 4

ALL THE ACTIVITIES AND GAMES IN *THE DYSCALCULIA TOOLKIT* LISTED ACCORDING TO THEIR NUMERACY TOPIC OR MAIN TEACHING POINTS

Use this table to help you find activities and games to address a particular need or misconception. Tracking sheets, based on this table, are provided amongst the online resources available via the Companion Website to help you make plans and programmes of work for individual pupils or groups and to record and track progress.

A downloadable version of this table is available via the Companion Website

NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Visual patterns for numbers up to 10	Make dot patterns for the numbers 1 to 10	Section 1
Visual patterns for numbers up to 5	Make transparent dot pattern cards for the numbers 1 to 5	Section 1
Visual patterns for numbers up to 5	Make 5 Game	Section 1
Visual patterns for numbers up to 6	Explore smaller numbers inside larger numbers	Section 1
Visual patterns for numbers up to 10	Change one dot pattern into another	Section 1
Visual patterns for numbers up to 10	Use Cuisenaire rods to learn all components to 10	Section 1
Visual patterns for numbers up to 10	Make a 'Story' of a number	Section 1
Visual patterns for numbers up to 10	Change dot patterns by adding or subtracting	Section 1
Visual patterns for numbers up to 12	Sort and re-sort a set of dominoes	Section 1
Visual patterns for numbers up to 20	Connect 'teen' numbers to those below 10	Section 2
Visual patterns for numbers up to 20	Focus on the 'teen' numbers	Section 2
Visual patterns for numbers up to 20	Explore 'teen' numbers with Cuisenaire rods	Section 2
Key components up to 10	Make dot patterns for the numbers 1 to 10	Section 1
Key components up to 10	Use dot patterns to explore odd and even	Section 1
Key components up to 10	Key Components Guessing Game	Section 1
Key components up to 10	Connect subtraction to addition	Section 1
Key components up to 10	Regroup: Apply logic to find new component facts	Section 1
Key components up to 10	Post-It Note Subtraction Game	Section 1
Doubles and near doubles up to 5 + 5	Make dot patterns for the numbers 1 to 10	Section 1
Doubles and near doubles up to 5 + 5	Use dot patterns to explore odd and even	Section 1
Doubles and near doubles up to 5 + 5	Regroup: Apply logic to find new component facts	Section 1
Doubles and near doubles up to 5 + 5	Explore and learn the doubles up to 5 + 5	Section 1
Doubles and near doubles	Use reasoning to find near-doubles	Section 1
Odd and even numbers up to 10	Use dot patterns to explore odd and even	Section 1
Odd and even numbers up to 10	Explore with Cuisenaire rods and with money	Section 1
Odd and even numbers up to 10	Odd and Even Collectors Game	Section 1
Numbers in relation to each other	Explore smaller numbers inside larger numbers	Section 1
Numbers in relation to each other	Numbers Inside Game	Section 1

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NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Numbers in relation to each other	Become familiar with Cuisenaire rods	Section 1
Numbers in relation to each other	Race to Tell a Story Game	Section 1
Numbers in relation to each other	Sort and re-sort a set of dominoes	Section 1
Components up to 5	Make 5 Game	Section 1
Components up to 5	Collect 5s Game	Section 1
Components up to 6	Explore smaller numbers inside larger numbers	Section 1
Components up to 6, 7, 8, 9 or 10	Clear the Deck Game	Section 1
Components up to 10	Sort and re-sort a set of dominoes	Section 1
Components up to 10	Regroup: Apply logic to find new component facts	Section 1
Components up to 10	Use Cuisenaire rods to learn all components to 10	Section 1
Components up to 10	Make a Story of a number	Section 1
Components up to 10	Find complements of 10 with Cuisenaire rods	Section 1
Components up to 10	Complementary addition	Section 1
Components up to 10	Use money for component work	Section 1
Components up to 12	Cover the Numbers / Shut the Box Game	Section 1
Complements to 10	Make a bead string	Section 1
Complements to 10	Learn complements of 10 on a bead string	Section 1
Complements to 10	How Many Beads? Game	Section 1
Complements to 10	Find complements of 10 with Cuisenaire rods	Section 1
Complements to 10	Ten in a Bed Game	Section 1
Complements to 10	Complements Number Search Game	Section 1
Complements to 10	Complements Ping-Pong Game	Section 1
Add/Subtract 1 or 2	Change dot patterns by adding or subtracting	Section 1
Add/Subtract 1 or 2	Focus on plus/minus 1 and plus/minus 2	Section 1
Add/Subtract 1 or 2	Who Has the Most Equations? Game	Section 1
Add 1, 2 or 3	Draw Your Race on a Number Line Game	Section 1
Add/Subtract small amounts	Collect 5s Game	Section 1
Add/Subtract small amounts	Numbers Inside Game	Section 1
Add/Subtract small amounts	Teach complementary addition	Section 1
Add/Subtract small amounts	Complementary addition on a number line	Section 1
Add/Subtract small amounts	Cover the Numbers / Shut the Box Game	Section 1
Missing numbers	Become familiar with Cuisenaire rods	Section 1
Missing numbers	Use Cuisenaire rods to learn all components to 10	Section 1
Missing numbers	Compare the difference and equalise	Section 1
Missing numbers	Hidden quantity subtraction	Section 1
Commutative property of addition	Make 5 Game	Section 1
Commutative property of addition	Collect 5s Game	Section 1
Commutative property of addition	Become familiar with Cuisenaire rods	Section 1

NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Commutative property of addition	Use Cuisenaire rods to learn all components to 10	Section 1
Commutative property of addition	Make and read equations with Cuisenaire rods	Section 1
Commutative property of addition	Cover the Numbers / Shut the Box Game	Section 1
Commutative property of addition	Learn complements to 10 with a bead string	Section 1
Commutative property of addition	Complements Number Search	Section 1
Commutative property of addition	Complements Ping-Pong Game	Section 1
Connect addition with subtraction	Change one dot pattern into another	Section 1
Connect addition with subtraction	Sort and re-sort a set of dominoes	Section 1
Connect addition with subtraction	Connect subtraction to addition	Section 1
Connect addition with subtraction	Regroup: Apply logic to find new component facts	Section 1
Connect addition with subtraction	Make and read equations with Cuisenaire rods	Section 1
Connect addition with subtraction	Draw and record equations in writing	Section 1
Connect addition with subtraction	Cover the Numbers / Shut the Box Game	Section 1
Connect addition with subtraction	Clear the Deck Game	Section 1
Connect addition with subtraction	Complements Number Search	Section 1
Connect addition with subtraction	Complements Ping-Pong Game	Section 1
Connect addition with subtraction	Ten in a Bed Game	Section 1
Connect addition with subtraction	Compare the difference and equalise	Section 1
Connect addition with subtraction	Post-It Note Subtraction Game	Section 1
Connect addition with subtraction	Hidden quantity subtraction	Section 1
Connect addition with subtraction	Teach complementary addition	Section 1
Connect addition with subtraction	Complementary addition on a number line	Section 1
Complementary addition below 10	Teach complementary addition	Section 1
Complementary addition below 10	Complementary addition on a number line	Section 1
Focus on the ‘teen’ numbers	Connect ‘teen’ numbers to those below 10	Section 2
Focus on the ‘teen’ numbers	Focus on the ‘teen’ numbers	Section 2
Focus on the ‘teen’ numbers	Explore ‘teen’ numbers with Cuisenaire rods	Section 2
Focus on the ‘teen’ numbers	Make a 20-step staircase	Section 3
Focus on the ‘teen’ numbers	It All Adds Up Game	Section 2
Focus on the ‘teen’ numbers	Locate 2-digit numbers	Section 2
Exchanging tens and units	Exchange units into tens	Section 3
Exchanging tens and units	Concrete counting on place value mats	Section 3
Exchanging tens and units	Magic 10s Game	Section 3
Complements to multiples of 10	Complements to 20	Section 2
Complements to multiples of 10	Cover 20 Game	Section 3
Complements to multiples of 10	Complements to larger multiples of 10	Section 2
Complements to multiples of 10	Complements on a number line	Section 2
Bridging through 5	How Many Beads? Game	Section 1
Bridging through 5	Five and What’s Left Game	Section 2
Bridging through 10	Introduce bridging with Cuisenaire rods	Section 2

(Continued)

(Continued)

NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Bridging through 10	Bridge through 10 on a number line	Section 2
Bridging through 10	Practise bridging and reinforce commutativity	Section 2
Bridging through 10	Frame an Addition Game	Section 2
Bridging through multiples of 10	Bridge through multiples of 10	Section 2
Bridging through multiples of 10	Race Along a Number Line and Bridge Game	Section 2
Bridging is not always necessary	Polka Dots Game	Section 2
Bridging is not always necessary	Race to the End of the Number Line Game	Section 2
Complementary addition 2-digit nos.	Complementary addition for subtraction	Section 2
Complementary addition 2-digit nos.	Frame a Subtraction Game	Section 2
Complementary addition 2-digit nos.	Subtracting round numbers	Section 2
Complementary addition 2-digit nos.	Harder complementary addition	Section 2
Partitioning 2-digit numbers	A flexible approach to partitioning	Section 2
Partitioning 2-digit numbers	Explore partitioning methods	Section 2
Partitioning 2-digit numbers	Partition numbers into tens and units	Section 3
Partitioning 2-digit numbers	Split off the 'teen' numbers	Section 3
Partitioning 2-digit numbers	Calculator Skittles Game	Section 3
Decomposition in subtraction	A flexible approach to partitioning	Section 2
Decomposition in subtraction	Avoid decomposition	Section 2
Decomposition in subtraction	Practise subtraction and decomposition	Section 3
Decomposition in subtraction	Spot the Decomposition Game	Section 3
Complements to 100	Complements to 100	Section 2
Complements to 100	Keep the Change! Game	Section 2
Complements to 100	Race to Cover 100 Game	Section 3
Doubling	Learn the doubles up to 10 + 10	Section 2
Doubling	Key fact: Double means 'multiply by 2'	Section 4
Doubling	Practise and extend the doubles facts	Section 2
Doubling	Double Take Game	Section 2
Doubling and halving	Halving is the opposite of doubling	Section 2
Doubling and halving	Find half of round numbers	Section 2
Doubling and halving	Function machines	Section 2
Doubling and halving	Key facts: ×5 is half of ×10	Section 4
Mental arithmetic strategies	*All work on components, complements and numerous other activities*	Sections 1–4
Mental arithmetic strategies	Regroup Game	Section 2
Mental arithmetic strategies	The Basic 8 Strategies	Section 2
Mental arithmetic strategies	Identify the best strategy for different situations	Sections 1–2
Derive new number facts by reasoning	Make dot patterns for the numbers 1 to 10	Section 1
Derive new number facts by reasoning	Change one dot pattern into another	Section 1
Derive new number facts by reasoning	Use dot patterns to explore odd and even	Section 1
Derive new number facts by reasoning	Collect 5s Game	Section 1
Derive new number facts by reasoning	Sort and re-sort a set of dominoes	Section 1

Introduction

This book is for teachers who are looking for practical ways to help pupils who struggle with numeracy. It is aimed mainly at primary teachers who do not have a specialist background in either maths or special needs.

I use the word 'teacher' loosely to mean anyone who supports children in their learning. Parents, for example, are ideally placed to use the practical activities and games in this book to promote maths as a practical subject full of patterns and puzzles, and therefore full of interest and fun. I hope the ideas in this book will be of interest to any adults who support pupils in junior school and in the early years of secondary school, no matter whether the adults in question are classroom teachers, teachers of numeracy, teaching assistants, parents or specialist staff in special needs departments. Because the suggestions presented here are designed to promote understanding and to help learners make mathematical connections, the ideas in this book can be used to teach the basic principles of numeracy to any learner.

What's new in this edition?

- Many more games. There are now 50 games in total, including twice as many games as previously provided in the section on basic calculation strategies (Section 2).
- Ten videos, with a total running time of 48 minutes, created specially for this new edition. On the CW you can find separate videos with commentary for each of the four sections of the book, plus six short stop-motion silent movies demonstrating a selection of games from every section.
- New pupil tracking sheets, organised by teaching points and by section. The tracking sheets can be edited before being printed off.
- More downloadable teaching materials. There are now 70 pages of resources, ready to be printed off for immediate use. They comprise 18 pages of teaching resources, 18 pages of activity sheets, 20 game boards, 6 pupil tracking sheets and 10 pages of summary tables to help readers identify which activities and games are designed to target different numeracy topics. The new material available for the first time in this edition includes teaching resources, activity sheets and game boards as well as the tracking sheets mentioned above.
- Easier access to the teaching resources. All the extra material accompanying the book is now accessible from the CW .

- New and improved illustrations. Extra illustrations have been added and all the existing illustrations have been re-formatted to improve consistency and clarity.
- An expanded list of recommended reading that now includes useful websites and links to various resources and suppliers.

What does this book contain?

Inside this book you will find a collection of teaching activities and games. The activities have been developed over a number of years of teaching dyslexic, dyspraxic and dyscalculic learners, either on a one-to-one basis or in small groups of pupils who have been withdrawn from lessons for extra support. The activities are equally appropriate for children who have been diagnosed as dyscalculic as for those whose difficulties with number arise from other specific learning difficulties such as dyslexia or dyspraxia.

The book is organised into four sections:

- Section 1: Early Number Work with Numbers Under 10
- Section 2: Basic Calculations with Numbers Above 10
- Section 3: Place Value
- Section 4: Times Tables, Multiplication and Division.

Resources for all four sections can be found on the CW for the book.

The Appendix, which can be found on the CW, contains a summary of the more commonly used concrete materials, including an introduction to Cuisenaire rods. Because I use Cuisenaire rods so extensively in my teaching activities, and have found so many people unfamiliar with their use, I have also included on the CW a leaflet of practical ideas, written originally for parents.

The CW provides easy access to 70 pages of useful teaching resources, including masters for dot pattern cards and for digit cards, base-10 resources, number tracks with Slavonic shading, a variety of activity sheets, two summary tables to help you choose activities and games to target particular numeracy topics, new pupil tracking sheets, as well as 20 game boards for all the board games in this book.

The philosophy behind my teaching methods is to provide children with the kinds of practical experiences that will help them build sound cognitive models. Because the emphasis is on doing the maths rather than recording it on paper, you will find very few worksheets or ideas for written work in this book. What you will find instead is more than 200 teaching activities and 50 games. I have deliberately included activities that require only what can easily be found in a normal maths classroom or can easily be acquired by parents, such as counters, Cuisenaire rods, Dienes blocks, digit cards, dice, dominoes, paper and pencils. There is no need to buy special equipment, or commercial games and resources that tend to target only a single topic. My activities are simple to set up and most are ready for immediate use with individual pupils or small groups; others just require copies of the games boards or activity sheets from the CW .

Why is there such a strong emphasis on games?

I invariably use a lot of games in my teaching, and not simply because they are fun. Provided that the games are carefully chosen, or carefully designed, to target only a single mathematical idea at a time, games furnish pupils with the opportunity and the incentive to practise the specific techniques that we want them to acquire, allowing the ideas to become habitual and gradually more fluent. For example, having taught pupils the complement facts of 10 – the five number bonds 5 + 5, 4 + 6, 3 + 7, 2 + 8 and 1 + 9 – by allowing the pupils to manipulate concrete materials as they explore these numerical relationships in an active and practical way, pupils will still need plenty of practice using these five facts before they can become absolutely secure. That is why you will find four games, as well as several activities, that specifically target the complement facts of 10. Learners with specific learning difficulties tend to need much more repetition and rehearsal than their peers, spread over a longer period. But there are only so many times someone can recite the facts, or complete worksheets featuring them, without boredom setting in.

In parallel with work on key facts, I regularly and explicitly teach pupils reasoning strategies about how to derive an unknown fact from a known and practised fact, such as how to find various steps of a multiplication table from the three key facts for that table. A worksheet is likely to give only mechanical practice in producing the answer because it has been designed as a way of testing automatic recall, whereas a well-designed game will practise the necessary steps required to reach an answer through logic and deduction.

As a teacher, I am constantly aware of the need to contrive situations in which the facts and techniques that my pupils need to master can be rehearsed in as many different ways as possible. Games are extremely valuable in this context because children are naturally motivated to spend time playing games over and over again and rarely notice how much learning and reinforcing is taking place while they are actively and productively engaged in play. As well as being more enjoyable, games are more powerful than worksheets because each time a game is played different challenges might be presented, in a different sequence, leading to new considerations and different outcomes, all of which provides for a much more varied, stimulating and active learning experience.

Purposeful activities and targeted games are central to my approach to teaching learners who have difficulties with basic maths. The activities I specify in this book are never intended to simply back up paper-and-pencil techniques or abstract methods. Neither are my games ever intended as just a bit of fun to fill in the spare time at the end of a lesson. All the activities and games that I design are invented to provide the actual learning experience for a variety of very specific maths topics.

How to use this book

Please do not feel that you ought to start at the beginning of the book and work through to the end, or even to keep to the sequence in which the ideas are presented. You should feel free to pick and choose activities, depending on your pupils and your knowledge of their particular areas of difficulty. Some activities may need to be repeated often, or revisited at regular intervals; others may be valuable to try only once for particular pupils, or not

at all. When activities naturally follow on from each other, the text clearly signals the fact. Some activities may need to be preceded by others from another section; for example, some understanding of place value (Section 3) is required before attempting some of the work on larger numbers (Section 2) and before some of the work on multiplication and division (Section 4). Once you begin working closely with pupils, you will find that you are the person best placed to uncover any misconceptions or sticking points that could usefully become the focus of subsequent lessons.

Each of the four sections starts with a short overview, putting the topic of that section into context. Following the overview, you will find a summary of the main problems associated with the topic leading to a list of ideas on how to help. These summaries are presented as bullet points for ease of reference. The remainder of each section is dedicated entirely to the teaching activities and games, set out as clearly and concisely as possible with a minimum of explanatory background or theory. Printable and photocopiable resources from all four sections are provided on the CW, making the activities and games accessible and ready to use, with the minimum of preparation.

I have targeted what I know to be specific areas of difficulty and have deliberately broken down the teaching and learning into extremely small steps. Each section is loosely structured in order of difficulty, starting with concrete activities and progressing gradually through learning activities that are designed to help pupils move through the intermediate diagrammatic stage and right up to the abstract stage of calculation.

Most of the activities are designed to be teacher-led, rather than for children to work through on their own. It is important to ask lots of questions, to direct the discussion carefully, to point out any connections with previous activities and other maths topics and to encourage pupils to talk a lot about what they are doing, and why, while they are doing it. Naturally, pupils will do best in an atmosphere where mistakes are regarded as a normal, and even an instructive, part of the learning process.

Whether inside the classroom or at home, the best results will be achieved by frequent, regular, short but unhurried, sessions, each of which should include a variety of activities and topics and a sensitive balance between revision and new content. Daily sessions will soon improve pupils' attitude and will steadily boost their self-assurance, their sense of achievement and their maths performance.

The 200+ activities are each labelled according to the main teaching point they have been designed to address. A list of teaching points is also included in the instructions for each of the 50 games. The main numeracy topic addressed by each game is summarised in the first of the two tables at the end of this Introduction. The second summary table at the end of this Introduction links the activities and games in this book to a list of numeracy topics and teaching points so that you can easily find ideas to target a particular gap in a pupil's knowledge or to address a specific misconception or need. As well as appearing at the end of this chapter, both the summary tables can be found on the CW . Tracking sheets, accessible from the CW, are provided for the first time in this latest edition of *The Dyscalculia Toolkit*. Closely linked to the summary tables, the tracking sheets are designed to help you plan work for individual students or groups of pupils, and to use for tracking and recording a learner's progress. The tracking sheets can be edited before being printed off.

What is dyscalculia?

Developmental dyscalculia was first recognised in the UK by the Department for Education and Science in 2001 and defined as: 'a condition that affects the ability to acquire arithmetical skills. Dyscalculic learners may have difficulty understanding simple number concepts, lack an intuitive grasp of numbers, and have problems learning number facts and procedures. Even if they produce a correct answer or use a correct method, they may do so mechanically and without confidence' (DfES 0512/2001, p. 2).

There is a debate about whether true dyscalculia differs from the maths difficulties experienced by some dyslexic and dyspraxic learners, a debate I am happy to leave to the academics. What matters to me is the fact that the same sorts of intervention seem to help many pupils who are underachieving in maths, whatever label they have been given. I believe that the coming years will see a growing recognition of the particular problems and educational needs of dyscalculic learners, in much the same way as the last three or four decades have seen an increasing acceptance of the existence of dyslexia and a developing consensus about the best teaching and learning approaches for these pupils.

Research into dyscalculia is still at an early stage, but it is estimated that dyscalculia affects roughly 4–6% of the population. This equates to at least one child in any average classroom.

What are the indicators for dyscalculia?

As a teacher, you might suspect that you have a dyscalculic pupil in your class if an otherwise competent student has a surprising level of difficulty with ordinary numeric operations and relies on finger-counting, often for all four arithmetic operations, well beyond the age at which most of the others in the class have progressed to more efficient strategies. A dyscalculic learner stands out as having no 'feel for numbers' at all, no ability to estimate even small quantities, and no idea whether an answer to an arithmetic problem is reasonable or not. Memory weaknesses, both long-term and short-term, are a great handicap and result in a pupil with dyscalculia being unable to remember facts and procedures accurately, or consistently, no matter how many times they try to learn them by heart. Pupils who have dyscalculia simply cannot remember their times tables reliably, and you may find they can recall some facts one day but not the next. They are also likely to lose track of what they are doing when attempting any procedure that requires more than two or three steps. Even basic counting can be a problem for pupils with dyscalculia, especially counting backwards.

Indicators for dyscalculia include:

- an inability to subitise (see without counting) even very small quantities
- an inability to estimate whether a numerical answer is reasonable
- weaknesses in both short-term and long-term memory
- an inability to count backwards reliably
- a weakness in visual and spatial orientation
- directional (left/right) confusion
- slow processing speeds when engaged in maths activities

- trouble with sequencing
- a tendency not to notice patterns
- a problem with all aspects of money
- a marked delay in learning to read a clock to tell the time
- an inability to manage time in daily life.

What about learners with other specific learning difficulties?

A dyslexic pupil might show many of the same indicators as those listed above, because it is thought that at least half of all dyslexics also have difficulties with maths. Outside the maths classroom, you might suspect that pupils are dyslexic if they read and write much less willingly and fluently than you would expect, if they read and re-read written material with little comprehension and if their spelling is particularly weak, inconsistent or bizarre. Dyslexic learners show much greater ability and understanding when speaking than you could ever guess from looking at the scrappy and minimal amount of written work they produce. Other indicators are memory weaknesses, problems with processing auditory information, and difficulties with planning and organisation.

A typical dyspraxic pupil does not seem to have the same long-term memory problems as a dyslexic and so might be able to remember times tables facts with ease. Dyspraxia, also known as DCD (developmental coordination disorder), mainly affects motor control, which results in pupils being clumsy and uncoordinated, poor at planning and organisation, and unsuccessful at subjects like PE and sports that require balance and coordination. Dyspraxic pupils cannot process sensory information properly and are therefore forever tripping and falling, dropping and breaking things, and mislaying their belongings. In the maths classroom, dyspraxic pupils have particular difficulty in handling equipment such as a ruler, a protractor or a set of compasses, and their written work is likely to be very messy and difficult to decipher.

A pupil with attention deficit hyperactivity disorder, may signal his (and it is usually a boy) presence by being unable to stop fidgeting or to sit still, being too easily distracted by outside stimuli, having a tendency to talk and interrupt excessively, and finding it extremely difficult to stay on task and see any undertaking through to the end. I mention the condition here only because nowadays pupils with ADHD or ADD tend to come under the umbrella term of 'learners with specific difficulties'. However, pupils with attention disorders may not have any specific problems with numeracy or maths once they have found a way to manage their impulsivity and concentration difficulties.

What kind of teaching do dyscalculic learners need?

All numeracy teaching should aim to help learners build up a sound mathematical understanding of numbers and their relationships. The basis of my own teaching approach with dyscalculic learners is to concentrate on numeracy and arithmetic, starting – crucially – with a variety of versatile concrete materials that provide practical experience and strongly visual models. Once a

numerical concept has been understood at the concrete level, then, and only then, will I begin to lead the learner gradually but steadily towards some of the more abstract and symbolic methods associated with higher level mathematics.

My own view is that a set of Cuisenaire rods is indispensable for working with dyscalculic learners. I find it the best, most versatile and most powerful tool to offer learners who are struggling to build a coherent mental model of the number system. I supplement Cuisenaire rods at the lower end with discrete items, such as counters or nuggets, that can be arranged and re-arranged into dot patterns for the numbers up to 10, and at the upper end with Dienes blocks or other base-10 equipment that can combine with Cuisenaire rods for concrete modelling of 3-digit numbers. One of the great strengths of Cuisenaire is that numbers are not presented as a collection of ones, so that the learner's focus is directed away from counting and towards number relationships. See the Appendix for an introduction to Cuisenaire rods and other concrete materials.

Dyscalculic learners, just like other learners, need to be able to count properly. Counting is, after all, the foundation of all numeracy. But, as soon as counting is secure, children have to be taught calculation strategies that do not rely on counting in ones. For this reason, I recommend plenty of work on building numbers from smaller components, splitting quantities up again into smaller chunks, and recombining the component pieces once more in order to fully explore the composition of numbers and the connection between addition and subtraction. The component work that I describe in this book is less static than simply learning the number bonds because its emphasis is on performing operations on numbers and seeing quantities change as a result of whatever action is being performed.

Working with chunks, or components, rather than ones, is the only antidote to the immature – and damaging – dependence on counting that is so common in pupils who struggle with numeracy. When counting is the only strategy known to learners, they have fallen into the 'counting trap'. See the Overviews of Section 1 and Section 2 for more about this pervasive problem. The only way out of the vicious cycle is to explicitly teach learners calculation strategies based on components, i.e. chunks, and not on counting on, or counting back, in ones.

Similarly, the times tables work and the multiplication and division activities that I recommend in this book are very far removed from the all-too-common practice of giving children a list of tables facts to learn by heart, a situation that leaves many pupils without any idea about what multiplication or division mean or how to use or apply the facts they have been asked to memorise. My teaching approach to tables is based on the area model of multiplication and division, an interpretation that inherently connects multiplication with division from the very beginning and one that can be modelled with Cuisenaire rods to produce rectangular shapes that are easy to read, understand and visualise.

Problems with numeracy often go hand in hand with significant memory weaknesses. This is why simple repetition will never be a way forward for dyscalculic pupils, however hard or often they are drilled. The best way to work around learners' memory problems is to focus on only a few key facts, those that are more important or have the widest application: first allow learners to thoroughly explore and internalise the key facts; then teach them explicitly how to derive whatever other facts they might need by reasoning logically from the key facts they already know.

Visualisation is a strategy that should be explicitly taught to dyscalculic pupils as a route towards mental calculation strategies. Immediately after a session of concrete work, pupils

can be asked to close their eyes and try to recreate some of the work in their mind's eye. Diagrammatic calculation methods, such as empty number lines for addition and subtraction or the area model for multiplication and division, can be introduced as a way of recording concrete work with Cuisenaire rods and later extended to support visualising techniques, thereby creating a bridge between concrete exploration with manipulative materials and the more abstract work that is the norm in mainstream schools. The transition between concrete and abstract work is an important stage that needs to be planned for and cannot be rushed.

My final observation about what kind of teaching approach works best for dyscalculic learners is a recommendation to break down every bit of teaching and learning into the tiniest of incremental steps and not to make any assumption about what pupils already know. For example, just because a child knows, say, that five counters can be arranged into the familiar dice pattern for 5, it does not follow that the same child will know that none of the other dice patterns can be created out of exactly five counters; or just because a child has discovered that adding 1 to each of the numbers up to 10 results in the next number in the counting sequence or that taking 1 away results in the previous number, it does not follow that the same child will know how to add 1 to a 2-digit number, let alone be able to work out how many to take away from a quantity in order to leave 1. Sound numerical understanding can only develop if it rests on secure foundations at every stage. The importance of tightly focused practical activities that address only a single new idea at a time cannot be overestimated.

What's next, after working through this book?

I have written two other books published by Sage that follow on from this one, although each book is complete in itself and can be used independently of the other two. *Overcoming Difficulties with Number* is aimed at learners who are working at a slightly higher level or who have, perhaps, already worked through many of the ideas in *The Dyscalculia Toolkit*. *Overcoming Difficulties with Number* analyses some key numeracy strategies – such as bridging through 10, or learning multiplication tables through the area model – in very great detail, setting out step-by-step instructions on how to teach the strategies to learners who find the concepts difficult. The focus is on teaching for understanding while at the same time helping learners make the transition from practical exploration to more abstract and canonical calculation methods.

The Dyscalculia Resource Book is a collection of ready-to-use and printable games and puzzles, all carefully targeted to practise the crucial foundation skills – such as adding and subtracting in component chunks rather than in ones, or deriving new numeracy facts from known key facts – that children who struggle with numeracy need to master before they can make any significant progress in maths. The games and puzzles in *The Dyscalculia Resource Book* are designed to reinforce what has previously been taught at a concrete level – for example through the activities and games in this *Dyscalculia Toolkit* book – and each is accompanied by clear instructions to the supervising adult on how to manage the activity so as to maximise the learning experience.

You are welcome to contact me through my website (www.ronitbird.com) with any feedback about how your children or pupils respond to the ideas in any of my books. On my website you will also find a list of Top Ten Tips for Parents, general information and online links to do with dyscalculia, a variety of free games and teaching resources, and details of my ebooks, all of which contain many demonstration videos.

SUMMARY OF THE MAIN TEACHING POINTS ADDRESSED BY THE GAMES IN *THE DYSCALCULIA TOOLKIT*

This table is provided because the name of each game does not always reveal exactly what topic it was designed to target (unlike the main teaching points of the activities, which are spelled out by their titles).

A downloadable version of this table is available via the Companion Website

GAME	MAIN NUMERACY TOPIC/TEACHING POINT	LOCATION
Make 5	Split and recombine numbers up to 5	Section 1
Numbers Inside	Identify smaller components of larger numbers	Section 1
Collect 5s	Add numbers up to 4 + 4	Section 1
Key Components Guessing Game	The key component facts: doubles or near-doubles	Section 1
Odd and Even Collectors	Recognise odd or even up to 10	Section 1
Draw Your Race on a Number Line	Empty number line versus number track	Section 1
Race to Tell a Story	Build the same target number in different ways	Section 1
Post-It Note Subtraction	Focus on subtraction, as inverse of addition	Section 1
Cover the Numbers / Shut the Box	Split and recombine numbers up to 12	Section 1
Clear the Deck	Components of a target number up to 10	Section 1
How Many Beads? How Many Hidden?	Bridging through 5 and through 10	Section 1
Complements Number Search	Complements to 10 (i.e. components of 10)	Section 1
Complements Ping-Pong	Complements to 10 (i.e. components of 10)	Section 1
Ten in a Bed	Complements to 10 (i.e. components of 10)	Section 1
Who Has the Most Equations?	Add and subtract 1 or 2	Section 1
Polka Dots	Visualise and use reasoning for components of 12	Section 2
Regroup	Visualise and use reasoning for numbers up to 20	Section 2
It All Adds Up	Build 'teen' numbers out of smaller components	Section 2
5 and What's Left	Bridging through 5 and through 10	Section 2
Frame an Addition	Bridging through 10	Section 2
Race Along a Number Line and Bridge	Bridging through 10	Section 2
Race to the End of the Number Line	Recognise when bridging is not needed	Section 2
Frame a Subtraction	Bridging through 10 for subtraction	Section 2
Keep the Change!	Complements to 100 (i.e. components of 100)	Section 2
Double Take	Doubling and reasoning about near-doubles	Section 2
Magic 10s	Grouping tens for place value	Section 3
Race to Cover 100	Exchange and decomposition	Section 3
Four Throws to Reach 100	Place value decimal system up to 100 or 1000	Section 3
Dice and spinner games	Place value, explored actively	Section 3
Spot the Decomposition	Decomposition in subtraction	Section 3
Win Counters on a 100-Square	Place value structure of 2-digit numbers	Section 3
Race Through a 100-Square	Place value structure of numbers up to 100	Section 3
Steer the Number	Place value structure of 2-digit numbers	Section 3
Two-Digit Sequences	Sequencing 2-digit numbers	Section 3
Three-Digit Sequences (Focus on Tens)	Sequencing 3-digit numbers	Section 3
Place Value Boxes	Place value of large numbers (6 digits)	Section 3

(Continued)

(Continued)

GAME	MAIN NUMERACY TOPIC/TEACHING POINT	LOCATION
Calculator Skittles	Place value of large numbers (4 digits or more)	Section 3
Jump 10	Adding 10 to a number	Section 3
The Six-Card Rounding Game	Place value in 2-digit numbers	Section 3
The Rounding Challenge	Place value in 2-digit or 3-digit numbers	Section 3
Don't Walk if You Can Take the Bus	Derive multiplication facts from key tables facts	Section 4
Mouse Tables	Learn and practise a chosen times table	Section 4
Self-correcting tables cards Game 1	Learn and practise a chosen times table	Section 4
Self-correcting tables cards Game 2	Finding the questions and focusing on division	Section 4
Self-correcting tables cards Game 3	Matching questions to answers in a chosen table	Section 4
Self-correcting tables cards Game 4	Snap, matching multiplication to division	Section 4
Multiples from the 1–6 Times Tables	Practise tables facts up to 6 × 6	Section 4
Products in a Row	Connect pairs of related multiplication tables	Section 4
Factors	Division: find factors of given multiples	Section 4
Areas on a Grid	The area model of multiplication and division	Section 4

ALL THE ACTIVITIES AND GAMES IN *THE DYSCALCULIA TOOLKIT* LISTED ACCORDING TO THEIR NUMERACY TOPIC OR MAIN TEACHING POINTS

Use this table to help you find activities and games to address a particular need or misconception. Tracking sheets, based on this table, are provided amongst the online resources available via the Companion Website to help you make plans and programmes of work for individual pupils or groups and to record and track progress.

A downloadable version of this table is available via the Companion Website

NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Visual patterns for numbers up to 10	Make dot patterns for the numbers 1 to 10	Section 1
Visual patterns for numbers up to 5	Make transparent dot pattern cards for the numbers 1 to 5	Section 1
Visual patterns for numbers up to 5	Make 5 Game	Section 1
Visual patterns for numbers up to 6	Explore smaller numbers inside larger numbers	Section 1
Visual patterns for numbers up to 10	Change one dot pattern into another	Section 1
Visual patterns for numbers up to 10	Use Cuisenaire rods to learn all components to 10	Section 1
Visual patterns for numbers up to 10	Make a 'Story' of a number	Section 1
Visual patterns for numbers up to 10	Change dot patterns by adding or subtracting	Section 1
Visual patterns for numbers up to 12	Sort and re-sort a set of dominoes	Section 1
Visual patterns for numbers up to 20	Connect 'teen' numbers to those below 10	Section 2
Visual patterns for numbers up to 20	Focus on the 'teen' numbers	Section 2
Visual patterns for numbers up to 20	Explore 'teen' numbers with Cuisenaire rods	Section 2
Key components up to 10	Make dot patterns for the numbers 1 to 10	Section 1
Key components up to 10	Use dot patterns to explore odd and even	Section 1
Key components up to 10	Key Components Guessing Game	Section 1
Key components up to 10	Connect subtraction to addition	Section 1
Key components up to 10	Regroup: Apply logic to find new component facts	Section 1
Key components up to 10	Post-It Note Subtraction Game	Section 1
Doubles and near doubles up to 5 + 5	Make dot patterns for the numbers 1 to 10	Section 1
Doubles and near doubles up to 5 + 5	Use dot patterns to explore odd and even	Section 1
Doubles and near doubles up to 5 + 5	Regroup: Apply logic to find new component facts	Section 1
Doubles and near doubles up to 5 + 5	Explore and learn the doubles up to 5 + 5	Section 1
Doubles and near doubles	Use reasoning to find near-doubles	Section 1
Odd and even numbers up to 10	Use dot patterns to explore odd and even	Section 1
Odd and even numbers up to 10	Explore with Cuisenaire rods and with money	Section 1
Odd and even numbers up to 10	Odd and Even Collectors Game	Section 1
Numbers in relation to each other	Explore smaller numbers inside larger numbers	Section 1
Numbers in relation to each other	Numbers Inside Game	Section 1

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NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Numbers in relation to each other	Become familiar with Cuisenaire rods	Section 1
Numbers in relation to each other	Race to Tell a Story Game	Section 1
Numbers in relation to each other	Sort and re-sort a set of dominoes	Section 1
Components up to 5	Make 5 Game	Section 1
Components up to 5	Collect 5s Game	Section 1
Components up to 6	Explore smaller numbers inside larger numbers	Section 1
Components up to 6, 7, 8, 9 or 10	Clear the Deck Game	Section 1
Components up to 10	Sort and re-sort a set of dominoes	Section 1
Components up to 10	Regroup: Apply logic to find new component facts	Section 1
Components up to 10	Use Cuisenaire rods to learn all components to 10	Section 1
Components up to 10	Make a Story of a number	Section 1
Components up to 10	Find complements of 10 with Cuisenaire rods	Section 1
Components up to 10	Complementary addition	Section 1
Components up to 10	Use money for component work	Section 1
Components up to 12	Cover the Numbers / Shut the Box Game	Section 1
Complements to 10	Make a bead string	Section 1
Complements to 10	Learn complements of 10 on a bead string	Section 1
Complements to 10	How Many Beads? Game	Section 1
Complements to 10	Find complements of 10 with Cuisenaire rods	Section 1
Complements to 10	Ten in a Bed Game	Section 1
Complements to 10	Complements Number Search Game	Section 1
Complements to 10	Complements Ping-Pong Game	Section 1
Add/Subtract 1 or 2	Change dot patterns by adding or subtracting	Section 1
Add/Subtract 1 or 2	Focus on plus/minus 1 and plus/minus 2	Section 1
Add/Subtract 1 or 2	Who Has the Most Equations? Game	Section 1
Add 1, 2 or 3	Draw Your Race on a Number Line Game	Section 1
Add/Subtract small amounts	Collect 5s Game	Section 1
Add/Subtract small amounts	Numbers Inside Game	Section 1
Add/Subtract small amounts	Teach complementary addition	Section 1
Add/Subtract small amounts	Complementary addition on a number line	Section 1
Add/Subtract small amounts	Cover the Numbers / Shut the Box Game	Section 1
Missing numbers	Become familiar with Cuisenaire rods	Section 1
Missing numbers	Use Cuisenaire rods to learn all components to 10	Section 1
Missing numbers	Compare the difference and equalise	Section 1
Missing numbers	Hidden quantity subtraction	Section 1
Commutative property of addition	Make 5 Game	Section 1
Commutative property of addition	Collect 5s Game	Section 1
Commutative property of addition	Become familiar with Cuisenaire rods	Section 1

NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Commutative property of addition	Use Cuisenaire rods to learn all components to 10	Section 1
Commutative property of addition	Make and read equations with Cuisenaire rods	Section 1
Commutative property of addition	Cover the Numbers / Shut the Box Game	Section 1
Commutative property of addition	Learn complements to 10 with a bead string	Section 1
Commutative property of addition	Complements Number Search	Section 1
Commutative property of addition	Complements Ping-Pong Game	Section 1
Connect addition with subtraction	Change one dot pattern into another	Section 1
Connect addition with subtraction	Sort and re-sort a set of dominoes	Section 1
Connect addition with subtraction	Connect subtraction to addition	Section 1
Connect addition with subtraction	Regroup: Apply logic to find new component facts	Section 1
Connect addition with subtraction	Make and read equations with Cuisenaire rods	Section 1
Connect addition with subtraction	Draw and record equations in writing	Section 1
Connect addition with subtraction	Cover the Numbers / Shut the Box Game	Section 1
Connect addition with subtraction	Clear the Deck Game	Section 1
Connect addition with subtraction	Complements Number Search	Section 1
Connect addition with subtraction	Complements Ping-Pong Game	Section 1
Connect addition with subtraction	Ten in a Bed Game	Section 1
Connect addition with subtraction	Compare the difference and equalise	Section 1
Connect addition with subtraction	Post-It Note Subtraction Game	Section 1
Connect addition with subtraction	Hidden quantity subtraction	Section 1
Connect addition with subtraction	Teach complementary addition	Section 1
Connect addition with subtraction	Complementary addition on a number line	Section 1
Complementary addition below 10	Teach complementary addition	Section 1
Complementary addition below 10	Complementary addition on a number line	Section 1
Focus on the ‘teen’ numbers	Connect ‘teen’ numbers to those below 10	Section 2
Focus on the ‘teen’ numbers	Focus on the ‘teen’ numbers	Section 2
Focus on the ‘teen’ numbers	Explore ‘teen’ numbers with Cuisenaire rods	Section 2
Focus on the ‘teen’ numbers	Make a 20-step staircase	Section 3
Focus on the ‘teen’ numbers	It All Adds Up Game	Section 2
Focus on the ‘teen’ numbers	Locate 2-digit numbers	Section 2
Exchanging tens and units	Exchange units into tens	Section 3
Exchanging tens and units	Concrete counting on place value mats	Section 3
Exchanging tens and units	Magic 10s Game	Section 3
Complements to multiples of 10	Complements to 20	Section 2
Complements to multiples of 10	Cover 20 Game	Section 3
Complements to multiples of 10	Complements to larger multiples of 10	Section 2
Complements to multiples of 10	Complements on a number line	Section 2
Bridging through 5	How Many Beads? Game	Section 1
Bridging through 5	Five and What’s Left Game	Section 2
Bridging through 10	Introduce bridging with Cuisenaire rods	Section 2

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NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Bridging through 10	Bridge through 10 on a number line	Section 2
Bridging through 10	Practise bridging and reinforce commutativity	Section 2
Bridging through 10	Frame an Addition Game	Section 2
Bridging through multiples of 10	Bridge through multiples of 10	Section 2
Bridging through multiples of 10	Race Along a Number Line and Bridge Game	Section 2
Bridging is not always necessary	Polka Dots Game	Section 2
Bridging is not always necessary	Race to the End of the Number Line Game	Section 2
Complementary addition 2-digit nos.	Complementary addition for subtraction	Section 2
Complementary addition 2-digit nos.	Frame a Subtraction Game	Section 2
Complementary addition 2-digit nos.	Subtracting round numbers	Section 2
Complementary addition 2-digit nos.	Harder complementary addition	Section 2
Partitioning 2-digit numbers	A flexible approach to partitioning	Section 2
Partitioning 2-digit numbers	Explore partitioning methods	Section 2
Partitioning 2-digit numbers	Partition numbers into tens and units	Section 3
Partitioning 2-digit numbers	Split off the 'teen' numbers	Section 3
Partitioning 2-digit numbers	Calculator Skittles Game	Section 3
Decomposition in subtraction	A flexible approach to partitioning	Section 2
Decomposition in subtraction	Avoid decomposition	Section 2
Decomposition in subtraction	Practise subtraction and decomposition	Section 3
Decomposition in subtraction	Spot the Decomposition Game	Section 3
Complements to 100	Complements to 100	Section 2
Complements to 100	Keep the Change! Game	Section 2
Complements to 100	Race to Cover 100 Game	Section 3
Doubling	Learn the doubles up to 10 + 10	Section 2
Doubling	Key fact: Double means 'multiply by 2'	Section 4
Doubling	Practise and extend the doubles facts	Section 2
Doubling	Double Take Game	Section 2
Doubling and halving	Halving is the opposite of doubling	Section 2
Doubling and halving	Find half of round numbers	Section 2
Doubling and halving	Function machines	Section 2
Doubling and halving	Key facts: ×5 is half of ×10	Section 4
Mental arithmetic strategies	*All work on components, complements and numerous other activities*	Sections 1–4
Mental arithmetic strategies	Regroup Game	Section 2
Mental arithmetic strategies	The Basic 8 Strategies	Section 2
Mental arithmetic strategies	Identify the best strategy for different situations	Sections 1–2
Derive new number facts by reasoning	Make dot patterns for the numbers 1 to 10	Section 1
Derive new number facts by reasoning	Change one dot pattern into another	Section 1
Derive new number facts by reasoning	Use dot patterns to explore odd and even	Section 1
Derive new number facts by reasoning	Collect 5s Game	Section 1
Derive new number facts by reasoning	Sort and re-sort a set of dominoes	Section 1

NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Derive new number facts by reasoning	Regroup: Apply logic to find new component facts	Section 1
Derive new number facts by reasoning	Use Cuisenaire to learn all components to 10	Section 1
Derive new number facts by reasoning	How Many Beads? Game	Section 1
Derive new number facts by reasoning	Find complements of 10 with Cuisenaire rods	Section 1
Derive new number facts by reasoning	Estimate and measure using Cuisenaire rods	Section 1
Derive new number facts by reasoning	The Regroup Game	Section 2
Derive new number facts by reasoning	Polka Dots Game	Section 2
Derive new number facts by reasoning	It All Adds Up Game	Section 2
Derive new number facts by reasoning	Connect 'teen' numbers to those below 10	Section 2
Derive new number facts by reasoning	Focus on the 'teen' numbers	Section 2
Derive new number facts by reasoning	Explore 'teen' numbers with Cuisenaire rods	Section 2
Derive new number facts by reasoning	A flexible approach to partitioning	Section 2
Derive new number facts by reasoning	Explore partitioning methods	Section 2
Derive new number facts by reasoning	Avoid decomposition in subtraction	Section 2
Derive new number facts by reasoning	9 is almost 10	Section 2
Derive new number facts by reasoning	Find near-complements and near-doubles	Sections 1–2
ENL (empty number lines)	Draw Your Race on a Number Line Game	Section 1
ENL (empty number lines)	Complementary addition on a number line	Section 1
ENL (empty number lines)	Locate 2-digit numbers in context	Section 2
ENL (empty number lines)	Complements on a number line	Section 2
ENL (empty number lines)	Bridge through 10 on a number line	Section 2
ENL (empty number lines)	Practise bridging and reinforce commutativity	Section 2
ENL (empty number lines)	Frame an Addition Game	Section 2
ENL (empty number lines)	Bridge through multiples of 10	Section 2
ENL (empty number lines)	Race Along the Number Line and Bridge Game	Section 2
ENL (empty number lines)	Race to the End of the Number Line Game	Section 2
ENL (empty number lines)	Complementary addition for subtraction	Section 2
ENL (empty number lines)	Frame a Subtraction Game	Section 2
ENL (empty number lines)	Subtracting round numbers	Section 2
ENL (empty number lines)	Harder complementary addition	Section 2
ENL (empty number lines)	Complements to 100	Section 2
ENL (empty number lines)	Jump 10 Game	Section 3
ENL (empty number lines)	Locate any number on a number line	Section 3
ENL (empty number lines)	Practise mental step-counting from given tables facts	Section 4
ENL (empty number lines)	Make times tables patterns on number lines	Section 4
Place value: 2- and 3-digit numbers	Concrete counting on place value mats	Section 3
Place value structure of 'teen' numbers	Make a 20-step staircase	Section 3
Place value structure of 'teen' numbers	Cover 20 Game	Section 3
Place value: 2-digit numbers	Exchange units into tens	Section 3
Place value: 2-digit numbers	Magic 10s Game	Section 3
Place value: 2-digit numbers	Race to Cover 100 Game	Section 3

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NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Place value: 2-digit numbers	Four Throws to Reach 100 Game	Section 3
Place value: 2-digit numbers	Win Counters on a 100-Square Game	Section 3
Place value: 2-digit numbers	Race Through a 100-Square Game	Section 3
Place value: 2-digit numbers	Steer the Number Game	Section 3
Place value: 2-digit numbers	Transform a 2-digit number in two steps	Section 3
Place value: 2-digit numbers	Two-Digit Sequences Game	Section 3
Place value: 2-digit numbers	Partition numbers into tens and units	Section 3
Place value: 2-digit numbers	Split off the 'teen' numbers	Section 3
Place value: 2-digit numbers	The Six-Card Rounding Game	Section 3
Place value: 2- or 3-digit numbers	Make and read numbers made of Cuisenaire rods or base-10	Section 3
Place value: 2- or 3-digit numbers	Dice and spinner games	Section 3
Place value: 2- or 3-digit numbers	Practise subtraction and decomposition	Section 3
Place value: 2- or 3-digit numbers	Spot the Decomposition Game	Section 3
Place value: 2- or 3-digit numbers	Practise adding / subtracting 10 and 100	Section 3
Place value: 2- or 3-digit numbers	Jump 10 Game	Section 3
Place value: 2- or 3-digit numbers	Locate any number on a number line	Section 3
Place value: 2- or 3-digit numbers	The Rounding Challenge Game	Section 3
Place value: 2- or 3-digit numbers	Teach × 10 and ÷ 10 as a shift between columns	Section 3
Place value: 2- or 3-digit numbers	Extend place value thinking to decimals	Section 3
Place value: 3-digit numbers	Build up large numbers, one column at a time	Section 3
Place value: 3-digit numbers	What is the value of …?	Section 3
Place value: 3-digit numbers	Three-Digit Sequences (Focus on Tens) Game	Section 3
Place value: decimal numbers	Teach ×10 and ÷10 as a shift between columns	Section 3
Place value: decimal numbers	Extend place value thinking to decimals	Section 3
Place value: decimal numbers	Connect decimal notation to money	Section 3
Place value: more than 3 digits	Use a spike abacus	Section 3
Place value: more than 3 digits	Teach the threefold repeating pattern	Section 3
Place value: more than 3 digits	Explore place value as a shorthand	Section 3
Place value: more than 3 digits	Read and write multi-digit numbers	Section 3
Place value: more than 3 digits	Place Value Boxes Game	Section 3
Place value: more than 3 digits	Calculator Skittles Game	Section 3
Place value: more than 3 digits	Teach ×10 and ÷10 as a shift between columns	Section 3
Place value: more than 3 digits	Extend place value thinking to decimals	Section 3
Place value: more than 3 digits	Connect decimal notation to money	Section 3
Multiplication as groups or arrays	Build small numbers out of equal-sized groups	Section 4
Multiplication as repeated addition	Connect step-counting with multiplication	Section 4
Multiplication as repeated addition	Step-count one or two steps from given facts	Section 4
Patterns created by tables facts	Make times tables patterns on a 100-square	Section 4

NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Patterns created by tables facts	Make times tables patterns on number lines	Section 4
Step-counting for multiplication facts	Build small numbers out of equal-sized groups	Section 4
Step-counting for multiplication facts	Connect step-counting with multiplication	Section 4
Step-counting for multiplication facts	Step-count one or two steps from given facts	Section 4
Step-counting for multiplication facts	Practise mental step-counting from given tables facts	Section 4
Area model of multiplication & division	Use Cuisenaire rods to show commutativity	Section 4
Area model of multiplication & division	Cuisenaire rods for multiplication and division	Section 4
Area model of multiplication & division	Key facts: ×5 is half of ×10	Section 4
Area model of multiplication & division	How many 10s? So, twice as many 5s	Section 4
Area model of multiplication & division	Find division facts by reasoning from key facts	Section 4
Area model of multiplication & division	×9 is almost ×10	Section 4
Area model of multiplication & division	Diagrammatic practice	Section 4
Area model of multiplication & division	Use rectangle sketches to derive new facts	Section 4
Area model of multiplication & division	Change the shape of the rectangle	Section 4
Area model of multiplication & division	Areas on a Grid Game	Section 4
Connect multiplication with division	Connect division to multiplication	Section 4
Connect multiplication with division	Diagrammatic practice	Section 4
Connect multiplication with division	Illustrate simple word problems	Section 4
Connect multiplication with division	Key facts: ×5 is half of ×10	Section 4
Connect multiplication with division	How many 10s? So, twice as many 5s	Section 4
Connect multiplication with division	Find division facts by reasoning from key facts	Section 4
Connect multiplication with division	Mouse Tables games	Section 4
Connect multiplication with division	Games using self-correcting cards	Section 4
Connect multiplication with division	Products in a Row Game	Section 4
Connect multiplication with division	Construct a multiplication grid	Section 4
Connect multiplication with division	Complete a partially-filled multiplication grid	Section 4
Connect multiplication with division	Multiples from the 1–6 Times Tables Game	Section 4
Connect multiplication with division	Factors Game	Section 4
Connect multiplication with division	Areas on a Grid Game	Section 4
Derive new tables facts by reasoning	Use Cuisenaire rods to show commutativity	Section 4
Derive new tables facts by reasoning	Step-count one or two steps from given facts	Section 4
Derive new tables facts by reasoning	Make times tables patterns on number lines	Section 4
Derive new tables facts by reasoning	How many 10s? So, twice as many 5s	Section 4
Derive new tables facts by reasoning	Find all the steps of any times table	Section 4
Derive new tables facts by reasoning	Find division facts by reasoning from key facts	Section 4
Derive new tables facts by reasoning	Practise all the steps of any times table	Section 4
Derive new tables facts by reasoning	Don't Walk If You Can Take the Bus Game	Section 4
Derive new tables facts by reasoning	Products in a Row Game	Section 4

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NUMERACY TOPIC/TEACHING POINT	ACTIVITY OR GAME	LOCATION
Derive new tables facts by reasoning	Harder mixed tables practice	Section 4
Derive new tables facts by reasoning	Factors Game	Section 4
Derive new tables facts by reasoning	Change the shape of the rectangle	Section 4
Derive new tables facts by reasoning	Use rectangle sketches to derive new facts	Section 4
Derive new tables facts by reasoning	Areas on a Grid Game	Section 4
Prepare for more advanced work	Boxes for long multiplication	Section 4